A කොටස

01 $U_r = 2^r + 1,\ r \geq 0$

$r=0$ විට,

$L.H.S = U_0 = 2$ $\qquad R.H.S = 2^0 + 1 = 2$ (5)

$\therefore L.H.S = R.H.S$

$\therefore r=0$ විට ප්‍රකාශනය සත්‍ය වේ.

$r=1$ විට,

$L.H.S = U_1 = 3$ $\qquad R.H.S = 2^1 + 1 = 3$ (5)

$\therefore L.H.S = R.H.S$

$\therefore r=1$ විට ප්‍රකාශනය සත්‍ය වේ.

$r = p$ සඳහා ප්‍රකාශනය සත්‍ය යැයි සිතමු.

$U_p = 2^p + 1$ — (i) $\qquad U_{p-1} = 2^{p-1} + 1$ — (ii)

$U_{p+1} = 3(2^p + 1) - 2U_{p-1}$ (5)

$U_{p+1} = 2.2^p + 1 + 2^p + 2 - 2U_{p-1}$

$U_{p+1} = 2^{p+1} + 1 + 2\underbrace{(2^{p-1} + 1 - U_{p-1})}_{0\ \because (ii)}$ (5)

$\therefore U_{p+1} = 2^{p+1} + 1$

$\therefore r = p+1$ විටද ප්‍රකාශනය සත්‍ය වේ. (5)

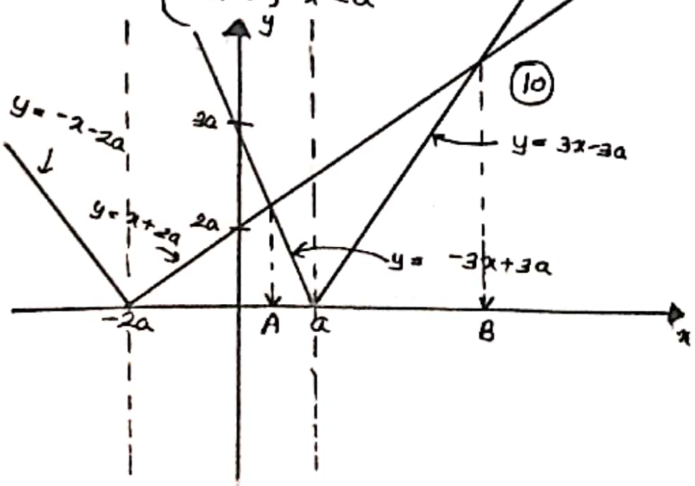
$\therefore$ ගණිත අභ්‍යුහන මූලධර්මයෙන් $\forall\ ; r \in \mathbb{Z}_0^+$ සඳහා ප්‍රකාශනය සත්‍ය වේ.

25

02 $|x+2a| > 3|x-a|$

$|x+2a| \begin{cases} x+2a; & x \geq -2a \\ -x-2a; & x < -2a \end{cases}$

$|x-a| \begin{cases} x-a; & x \geq a \\ -x+a; & x < a \end{cases}$



A =>

$x + 2a = -3x + 3a$

$x = \frac{a}{4}$ (5)

B =>

$x + 2a = 3x - 3a$

$2x = 5a$

$x = \frac{5a}{2}$ (5)

$\therefore$ විසඳුම් කුලකය,

$\left\{x ;\ x \in \mathbb{R} ;\ \frac{a}{4} < x < \frac{5a}{2}\right\}$ (5)

25

03 $f(x) = 2\lambda x^2 + 2(\lambda+4)x + 9\ ;\ (\lambda \neq 0)$

$f(x) > 0$ නම්,

$2\lambda > 0$

$\lambda > 0$ — (i) (5)

$\Delta_x < 0$

$[2(\lambda+4)]^2 - 4(2\lambda)9 < 0$ (5)

$4(\lambda^2 + 8\lambda + 16 - 18\lambda) < 0$

$\lambda^2 - 10\lambda + 16 < 0$

$(\lambda - 8)(\lambda - 2) < 0$ (5)

$\xrightarrow{\quad (+)\quad 2 \quad (-) \quad 8 \quad (+) \quad} \lambda$

$2 < \lambda < 8$ — (ii) (5)

(i) හා (ii) න්,

$2 < \lambda < 8$ (5)

25

04 $ax^2 + bx + c = 0 <^{\alpha}_{\beta}$

$\alpha + \beta = -\frac{b}{a}$ (5) $\qquad \alpha\beta = \frac{c}{a}$

$a(x-1)^2 + b(x-1)(x-2) + c(x-2)^2 = 0$

$(a+b+c)x^2 - (2a+3b+4c)x + (a+2b+4c) = 0 <^{p}_{q}$

$p + q = \frac{2a+3b+4c}{a+b+c}$

$= \frac{2 + 3(\frac{b}{a}) + 4(\frac{c}{a})}{1 + (\frac{b}{a}) + (\frac{c}{a})}$

$= \frac{2 - 3(\alpha+\beta) + 4\alpha\beta}{1 - (\alpha+\beta) + \alpha\beta}$

$= \frac{(1-2\alpha)(1-\beta) + (1-2\beta)(1-\alpha)}{(1-\alpha)(1-\beta)}$

$= \frac{(1-2\alpha)}{(1-\alpha)} + \frac{(1-2\beta)}{(1-\beta)}$ (5)

$pq = \frac{a+2b+4c}{a+b+c}$

$= \frac{1 + 2\frac{b}{a} + 4\frac{c}{a}}{1 + \frac{b}{a} + \frac{c}{a}}$

$$= \frac{1-2(\alpha+\beta)+4\alpha\beta}{1-(\alpha+\beta)+\alpha\beta}$$

$$= \frac{(1-2\alpha)(1-2\beta)}{(1-\alpha)(1-\beta)} \quad 5$$

$\therefore$ මූල දෙක $= \dfrac{1-2\alpha}{1-\alpha}, \ \dfrac{1-2\beta}{1-\beta}$

5 5

$\boxed{25}$

$\boxed{05}$ $f(x) = \begin{cases} \dfrac{x^2-1}{x^2-2|x-1|-1} & ; x \neq 1 \end{cases}$

$|x-1| \begin{cases} x-1 & ; x \geq 1 \\ 1-x & ; x < 1 \end{cases}$

$$\lim_{x\to 1^+} f(x) = \frac{x^2-1}{x^2-2(x-1)-1}$$

$$\lim_{x\to 1^+} f(x) = \frac{(x-1)(x+1)}{(x-1)^2} \to \frac{2}{0} \to \infty \quad 5$$

$$\lim_{x\to 1^-} f(x) = \frac{x^2-1}{x^2-2(1-x)-1}$$

$$= \frac{(x-1)(x+1)}{(x+3)(x-1)} \quad 5$$

$$= \frac{2}{4}$$

$$= \frac{1}{2} \quad 5$$

$x=1$ දී ; $f(x) = \frac{1}{2}$

$\lim\limits_{x\to 1^-} f(x) \neq \lim\limits_{x\to 1^+} f(x)$ නිසා, 5

$x=1$ දී ; $f(x)$ සන්තතික නොවේ. 5

$\boxed{25}$

$\boxed{06}$ $$\lim_{x\to 0} \frac{[(a-n)nx - \tan x]\sin nx}{x^2} = 0$$

$$\lim_{x\to 0}\left[\frac{(a-n)\,nx\sin(nx)}{x^2} \ (5) - \frac{\tan x \sin(nx)}{x^2} \ (5)\right] = 0$$

$$n^2(a-n)\left(\lim_{x\to 0}\frac{\sin(nx)}{nx}\right) - \lim_{x\to 0}\left(\frac{\sin x \cdot \sin(nx)}{x^2 \cdot \cos x}\right) = 0$$

$$n^2(a-n) - n\left(\lim_{x\to 0}\underbrace{\frac{\sin x}{x}}_{=1}\right)\cdot\left(\lim_{x\to 0}\underbrace{\frac{\sin nx}{nx}}_{=1}\right)\cdot\left(\lim_{x\to 0}\frac{1}{\cos x}\right) = 0$$

5 5

$$n^2(a-n) - n = 0$$

$$n(na-n^2-1) = 0$$

$$a = n + \frac{1}{n} \quad 5$$

$\boxed{25}$

$\boxed{07}$ $x^3 - y^2 = 0$

$$3x^2 - 2y\cdot\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{3x^2}{2y}$$

$$\left(\frac{dy}{dx}\right)_{(4t^2,\,8t^3)} = \frac{3(4t^2)^2}{2\cdot 8t^3} = 3t \quad 5$$

$\therefore$ ස්පර්ශකයේ සමීකරණය,

$$(y-8t^3) = 3t(x-4t^2) \quad \text{---(i)}$$

$$y - 3tx + 4t^3 = 0 \quad 5$$

ස්පර්ශකය, නැවත $x = t_0^2$ දී වක්‍රය හමුවේ යැයි, සිතමු.

එවිට, $y = t_0^3$ වේ.

x හා y ආදේශයෙන් (i) හි දී,

$$(t_0^3 - 8t^3) = 3t(t_0^2 - 4t^2)$$

$$(t_0-2t)(t_0^2+4tt_0+4t^2) = 3t(t_0-2t)(t_0+2t)$$

$$(t_0-2t)(t_0^2+4tt_0+4t^2-3tt_0-6t^2) = 0$$

$$(t_0-2t)(t_0^2+tt_0-2t^2) = 0$$

$$(t_0-2t)(t_0+2t)(t_0-t) = 0 \quad 5$$

$t_0 = -2t$ විට, $\quad t = t$ විට

$x = t_0^2 = 4t^2 \quad x = t^2$

$y = t_0^3 = 8t^3 \quad y = t^3$

$Q = (4t^2, 8t^3)$ හෝ $Q = (t^2, t^3)$

5 5

$\boxed{25}$

$\boxed{08}$ (i) අඳුරු කළ වර්ගඵලය A නම්,

$$A = \int_2^3 \frac{6}{(2x-3)}\,dx$$

$$A = 3\int_2^3 \frac{2}{(2x-3)}\,dx$$

$$A = 3\left(\ln|2x-3|\right)_2^3 \quad 5$$

$$= 3\left[\ln|2.3-3| - \ln|2.2-3|\right]$$

$A = 3\ln 3$ වර්ග ඒකක 5

(ii) $V = \int_2^3 \pi y^2 dx$

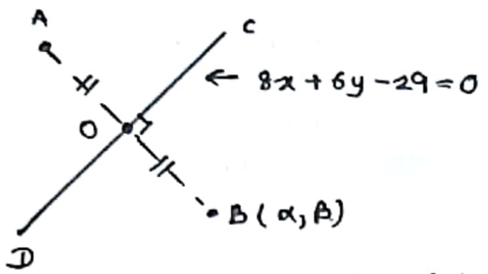
$V = \pi \int_2^3 \frac{36}{(2x-3)^2} dx$ (5)

$V = -18\pi \left[\frac{1}{2x-3}\right]_2^3 = -18\pi \left[\frac{1}{3} - 1\right]$ (5)

$\underline{\underline{V = 12\pi}}$ ඝන ඒකක (5)

$\boxed{25}$

$\boxed{09}$



$O \equiv \left(\frac{\alpha-1}{2}, \frac{\beta+2}{2}\right)$

O ලක්ෂ්‍යය ලම්බ සමච්ඡේදකයේ සමීකරණය තෘප්ත කරයි. (5)

$8\left(\frac{\alpha-1}{2}\right) + 6\left(\frac{\beta+2}{2}\right) - 29 = 0$

$4\alpha + 3\beta = 27$ —(i) (5)

$m_{AB} = \frac{\beta-2}{\alpha+1}$ $\qquad m_{CD} = \frac{-4}{3}$

$m_{AB} \cdot m_{CD} = -1$ (5)

$-\frac{4}{3} \cdot \frac{\beta-2}{\alpha+1} = -1$

$4\beta - 8 = 3\alpha + 3$

$4\beta - 3\alpha = 11$ —(ii)

(i) $\times 3$ + (ii) $\times 4$ =>

$25\beta = 125$

$\underline{\underline{\beta = 5}}$ (5) $\qquad \underline{\underline{\alpha = 3}}$ (5)

$\boxed{25}$

$\boxed{10}$ $\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$

$\left.\begin{array}{l} \sin^{-1}\left(\frac{5}{x}\right) = \alpha \Rightarrow \sin\alpha = \frac{5}{x} \\ \sin^{-1}\left(\frac{12}{x}\right) = \beta \Rightarrow \sin\beta = \frac{12}{x} \end{array}\right\}$ (5)

$\alpha + \beta = \frac{\pi}{2}$ (5)

$\alpha = \frac{\pi}{2} - \beta$

$\sin\alpha = \sin\left(\frac{\pi}{2} - \beta\right)$

$\sin\alpha = \cos\beta$

$\frac{5}{x} = \sqrt{1 - \sin^2\beta}$

$\frac{5}{x} = \sqrt{1 - \left(\frac{12}{x}\right)^2}$ (5)

$\frac{25}{x^2} = \frac{x^2 - 144}{x^2}$

$x^2 = 169$ (5)

$x = \pm 13$

$\cancel{x = -13}$ $\qquad \underline{\underline{\therefore x = 13}}$ (5)

$\boxed{25}$

B කොටස

$\boxed{11}$ (a) $k \in \mathbb{R}$ $\quad p(x) = x^2 - 2x + k \ ; \ (x \in \mathbb{R})$

(i) $p(x) < 0 \Rightarrow$

$p(x) = \underbrace{(x-1)^2}_{\geq 0} + (k-1)$ (5)

$p(x) < 0$ නම්; $k-1 < 0$ විය යුතුයි. $\therefore k < 1$ (5)

k නිඛිලමය අගය $= 0$ (5)

(ii) $p(x) = (1-k)x$

$x^2 - 2x + k = (1-k)x$

$x^2 + (k-3)x + k = 0$

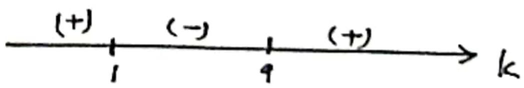
මූල තාත්වික නම්,

$\Delta_x \geq 0$

$(k-3)^2 - 4(1)k \geq 0$ (5)

$k^2 - 10k + 9 \geq 0$

$(k-9)(k-1) \geq 0$ (5)



$k \leq 1$ හෝ $k \geq 9$ (5)

∴ අවශ්‍ය කුලකය

$\underline{\underline{\{k ; k \in \mathbb{R} \ \ k \leq 1 \text{ හෝ } k \geq 9\}}}$ (5)

(ii) $f(x) = x^2 - 2x + k < \begin{matrix}\alpha \\ \alpha^2\end{matrix}$

$$\left.\begin{aligned}\alpha + \alpha^2 &= 2 \quad \text{— (i)} \\ \alpha \cdot \alpha^2 &= k \\ \alpha^3 &= k \quad \text{— (ii)}\end{aligned}\right\} (5) \qquad \begin{aligned}\alpha(\alpha+1) &= 2 \\ \alpha^3(\alpha+1)^3 &= 2^3\end{aligned}$$

$$\alpha^3\left[\alpha^3 + 3(\alpha^2+\alpha) + 1\right] = 8 \quad (5)$$

$$k\,(k + 3.(2) + 1) = 8$$

$$k^2 + 7k - 8 = 0 \quad (5)$$

$$(k+8)(k-1) = 0$$

(5) $k = 1$ හෝ $k = 8$ (5)

$\boxed{60}$

(b) $f(x) = (\alpha x - \beta)\, g(x) + R$

$x = \frac{\beta}{\alpha}$ විට,

$$f\left(\frac{\beta}{\alpha}\right) = \left(\alpha \cdot \frac{\beta}{\alpha} - \beta\right) g(x) + R$$

$$R = f\left(\frac{\beta}{\alpha}\right) \quad (5)$$

$$f(x) = (x^2 - \alpha^2)\, h(x) + (Ax + B)$$

$$f(x) = (x-\alpha)(x+\alpha)\, h(x) + (Ax + B)$$

$x = \alpha$

$f(\alpha) = A\alpha + B$ — (i) (5)

$x = -\alpha$

$f(-\alpha) = -A\alpha + B$ — (ii) (5)

(i) − (ii) $\Rightarrow$

$$f(\alpha) - f(-\alpha) = 2A\alpha$$

$$A = \frac{f(\alpha) - f(-\alpha)}{2\alpha} \quad (5)$$

(i) + (ii), $\Rightarrow$

$$f(\alpha) + f(-\alpha) = 2B$$

$$B = \frac{f(\alpha) + f(-\alpha)}{2} \quad (5)$$

$f(x) = (x^2 - 4)\, t(x) + (Ax + B)$ (5)

හෝ, $\alpha = 2$ ට.

$$\therefore \text{ශේෂය} = \frac{f(2) - f(-2)}{2(2)} + \frac{f(2) + f(-2)}{2}$$

$$= \frac{(2^4 + 5(2)^2 - 36) - ((-2)^4 + 5(-2)^2 - 36)}{4}\, x$$

(5) (5)

$$+ \frac{(2^4 + 5(2)^2 - 36) - ((-2)^4 + 5(-2)^2 - 36)}{4}$$

(5)

$$= (0)\,x + \frac{2^5 + 10.2^2 - 72}{4}$$

$$= 0 \quad (5)$$

$$\therefore x^4 + 5x^2 - 36 = (x^2 - 4)(x^2 + Bx + C)$$

x^2 සං

$5 = C - 4$

$C = 9$

x සං

$-4B = 0$

$B = 0$

$$\therefore x^4 + 5x^2 - 36 = (x^2 - 4)(x^2 + 9)$$

$$= (x-2)(x+2)(x^2+9) \quad (5)$$

$$f(x) = 0$$

$$(x-2)(x+2)(x^2+9) = 0$$

$x - 2 = 0$ හෝ $x + 2 = 0$ හෝ $x^2 + 9 = 0$

$\therefore$ තාත්වික මූල $= -2, 2$ (5)

තාත්වික මූල දෙකක් පමණක් ඇත.

$\boxed{60}$

(c) (i) $\log_a x = y$ ලෙස ගනිමු.

$$x = a^y$$

දෙපසම $\log_b$ ගැනීමෙන්.

$$\log_b x = \log_b a^y$$

$$\log_b x = y \log_b a$$

$$y = \frac{\log_b x}{\log_b a}$$

$$\therefore \log_a x = \frac{\log_b x}{\log_b a} \quad (5)$$

(ii) $x = x^m$ ද, $a = a^n$ ද ආදේශයෙන්,

$$\log_{a^n} x^m = \frac{\log_b x^m}{\log_b a^n} = \frac{m}{n} \frac{\log_b x}{\log_b a}$$

(5) (5)

(iii) $\log_a x + \log_{a^2} x^2 + \log_{a^3} x^3 + \log_{a^4} x^4 + \ldots + \log_{a^{2021}} x^{2021}$ (5)

$= \log_a x + \frac{2}{2}\log_a x + \frac{3}{3}\log_a x + \ldots + \frac{2021}{2021}\log_a x$

$= \log_a x + \log_a x + \ldots\ldots + \log_a x$

$= 2021 \log_a x$ (5)

$= \log_a x^{2021}$

$\boxed{30}$

$\boxed{12}$

(c) $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$

(i) $A^2 - 4A - 5I$

$= \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} - 4\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} - 5\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{pmatrix} - \begin{pmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{pmatrix} - \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ (10)

$\therefore A^2 - 4A - 5I = 0$

(ii) $A^2 - 4A - 5I = 0$

$\times A^{-1}$

$\underbrace{A^{-1}.A}_{=I}.A - 4.\underbrace{A^{-1}.A}_{=I} - 5.A^{-1}I = 0$ (5)

$IA - 4I - 5IA^{-1} = 0$

$A^{-1} = -(4I - A).\frac{1}{5}$

$A^{-1} = \frac{1}{5}(A - 4I)$

$= -\frac{1}{5}\left[4\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}\right]$

$= \frac{1}{5}\begin{pmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{pmatrix}$ (5)

(iii) $\underbrace{\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}}_{A}\underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} 6 \\ 7 \\ 7 \end{pmatrix}}_{Y}$ (5)

$AX = Y$

$A^{-1}.A.X = A^{-1}.Y$

$X = A^{-1}.Y$

$X = \frac{1}{5}\begin{pmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{pmatrix}\begin{pmatrix} 6 \\ 7 \\ 7 \end{pmatrix}$

$X = \frac{1}{5}\begin{pmatrix} -18+14+14 \\ 12-21+14 \\ 12+14+21 \end{pmatrix}$

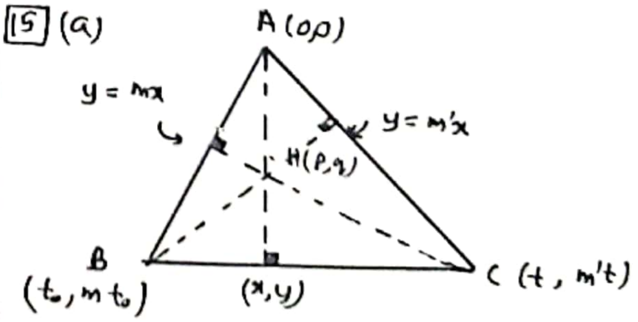
$X = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ (5)

$\therefore x = 2,\ y = 1,\ z = 1$

(5) (5)

$\boxed{40}$

$\boxed{15}$ (a)



$A \equiv (0,0)$ (5)

$$\left(\frac{m't-q}{t-p}\right).m = -1 \quad (5)$$

$$mm't - mq = p - t$$

$$t(1+mm') = p+qm$$

$$t = \frac{p+qm}{1+mm'} \quad (5)$$

එලෙසම, $t_0 = \dfrac{p+qm'}{1+mm'}$ (5)

$$\frac{(m't - mt_0)}{(t-t_0)} = \frac{y-mt_0}{x-t_0} \quad (5)$$

$$\frac{m'\left(\frac{p+qm}{1+mm'}\right) - m\left(\frac{p+qm'}{1+mm'}\right)}{\frac{p+qm-p-qm'}{(1+mm')}} = \frac{y-m\left(\frac{p+qm'}{1+mm'}\right)}{x-\left(\frac{p+qm'}{1+mm'}\right)}$$

$$-\frac{(m'-m)p}{(m'-m)q} = \frac{y-m\left(\frac{p+qm'}{1+mm'}\right)}{x-\left(\frac{p+qm'}{1+mm'}\right)} \quad (5) \quad (10)$$

$$px+qy = \frac{(p+qm)(p+qm')}{(1+mm')} \quad (5)$$

$\boxed{50}$

$$BC \text{ දුර} = \sqrt{(t-t_0)^2+(m't-mt_0)^2} \quad (5)$$

$$= \sqrt{\left(\frac{p+qm}{1+mm'} - \frac{p+qm'}{1+mm'}\right)^2 + \left(m'\cdot\frac{p+qm}{1+mm'} - m\frac{p+qm'}{1+mm'}\right)^2}$$

$$= \sqrt{\frac{q^2(m-m')^2}{(1+mm')^2} + \frac{p^2(m-m')^2}{(1+mm')^2}} \quad (5)$$

$$= \left|\frac{m-m'}{1+mm'}\right|\sqrt{p^2+q^2} \quad (5)$$

$\tan\theta = m$

$\tan\alpha = m'$

$$\alpha + \frac{\pi}{4} = \theta$$

$$\frac{\pi}{4} = \theta - \alpha$$

$$\tan\frac{\pi}{4} = \tan(\theta-\alpha)$$

$$1 = \frac{\tan\theta - \tan\alpha}{1+\tan\theta\tan\alpha}$$

$$1 = \frac{m-m'}{1+mm'} \quad (5)$$

$$\therefore BC = \sqrt{p^2+q^2} \quad (5)$$

$$AH = \sqrt{(p-0)^2+(q-0)^2} = \sqrt{p^2+q^2} \quad (5)$$

$\therefore B\hat{A}C = \frac{\pi}{4}$ නම්, $BC = AH$ වේ. (5)

$\boxed{35}$

(b) $A(0,1)$, $B(0,-1)$

$$S \equiv x^2+y^2+2gx+2fy+c=0$$

$A(0,1)$,

$$0^2+1^2+2g(0)+2f(1)+c=0 \quad (5)$$

$$1+2f+c=0 \quad \text{—(i)}$$

$B(0,-1)$,

$$0^2+(-1)^2+2g(0)+2f(-1)+c=0 \quad (5)$$

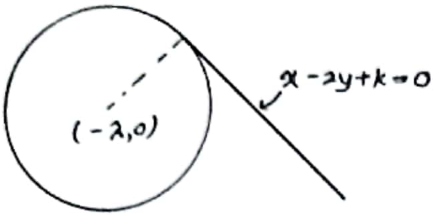
$$1-2f+c=0 \quad \text{—(ii)}$$

(i) −(ii)

$\underline{f=0}$ $\quad$ $\underline{c=-1}$ (5)

$$\therefore S \equiv x^2+y^2+2gx-1=0$$

$\therefore$ එය $x^2+y^2+2\lambda x-1=0$ ආකාරයෙන් වෘත්ත කුලකයකි. (5)



$$\frac{-\lambda-2(0)+k}{\sqrt{1^2+(-2)^2}} = \sqrt{(-\lambda)^2+1} \quad 10$$

$$\frac{k-\lambda}{\sqrt{5}} = \sqrt{\lambda^2+1} \quad 5$$

$$k^2-2k\lambda+\lambda^2 = 5(\lambda^2+1)$$

$$4\lambda^2+2k\lambda+5-k^2 = 0 \text{ — (i)} \quad \nearrow \lambda_1, \ \searrow \lambda_2$$

$$\Delta_\lambda > 0 \quad 5$$

$$(2k)^2 - 4.(4)(5-k^2) > 0$$

$$k^2 - 20 + 4k^2 > 0 \quad 5$$

$$\underline{\underline{k^2 > 4}}$$

එවැනි වෘත්ත දෙකේ සමීකරණය,

$$x^2+y^2+2\lambda_1 x-1=0$$

$$x^2+y^2+2\lambda_2 x-1=0 \text{ ලෙස ගනිමු.}$$

ප්‍රලම්බව ඡේදනය වෙයි නම්,

$$2gg_1+2ff_1 = c_1+c_2 \quad 5$$

$$2\lambda_1\lambda_2 = -2$$

$$\lambda_1\lambda_1 = -1 \text{ — (ii)} \quad 5$$

(i) න්,

$$\lambda_1\lambda_2 = \frac{5-k^2}{4} \quad 5$$

$$-1 = \frac{5-k^2}{4}$$

$$k^2 = 9$$

$$\underline{\underline{k = \pm 3}} \quad 5$$

$\boxed{65}$

$\boxed{16}$

(a) $4\sin\left\{x+\frac{\pi}{3}\right\}\cos\left\{x-\frac{\pi}{6}\right\} = \lambda^2+\sqrt{3}\sin 2x-\cos 2x$

$$2\left[\sin\left\{2x+\frac{\pi}{6}\right\}+\sin\frac{\pi}{2}\right] = \lambda^2+\sqrt{3}\sin 2x-\cos 2x$$
5

$$2+2\sin\left\{2x+\frac{\pi}{6}\right\} = \lambda^2+2\left(\sin 2x\,\frac{\sqrt{3}}{2}-\cos 2x.\frac{1}{2}\right)$$
5

$$2+2\sin\left\{2x+\frac{\pi}{6}\right\} = \lambda^2+2\sin\left\{2x-\frac{\pi}{6}\right\}$$

$$2\left[\sin\left\{2x+\frac{\pi}{6}\right\}-\sin\left\{2x-\frac{\pi}{6}\right\}\right] = \lambda^2-2$$

$$\underline{\underline{\cos 2x = \frac{\lambda^2-2}{2}}} \quad 5$$

$$-1 \le \cos 2x \le 1$$

$$-1 \le \frac{\lambda^2-2}{2} \le 1$$

$$-2 \le \lambda^2-2 \le 2 \quad 5$$

$$0 \le \lambda^2 \le 4$$

$$(\lambda-2)(\lambda+2) \le 0 \quad 5$$

(−) −2 (−) 2 (+) λ

$$\underline{\underline{-2 \le \lambda \le 2}} \quad 5$$

$\boxed{35}$

(b) $\tan 3\theta = 1$

$$\tan 3\theta = \tan\frac{\pi}{4}$$

$$3\theta = n\pi+\frac{\pi}{4} \ : n \in \mathbb{Z}$$

$$\underline{\underline{\theta = \frac{n\pi}{3}+\frac{\pi}{12}}} \quad 5$$

$n=0$; $\theta=\frac{\pi}{12}$, $n=1$; $\theta=\frac{5\pi}{12}$ $n=2$; $\theta=\frac{3\pi}{4}$

$$\tan 3\theta = 1 \quad 5$$

$$\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta} = 1$$
5

$$\tan^3\theta-3\tan^2\theta-3\tan\theta+1 = 0$$

$x = \tan\theta$ විට;

$$x^3-3x^2-3x+1=0 \quad 5$$

මූලයන්, $\tan\frac{\pi}{12}$, $\tan\frac{5\pi}{12}$, $\tan\frac{3\pi}{4}$ වේ. 5

$(x+1)(x^2-4x+1)=0$ $\quad x^2-4x+1=0 \nearrow \tan\frac{\pi}{12}$, $\searrow \tan\frac{5\pi}{12}$

$x+1=0$

$x=-1$

$x=\tan\frac{3\pi}{4}$

$(x-2)^2-3=0$

$(x-2-\sqrt{3})(x-2+\sqrt{3})=0$

$x=2+\sqrt{3}$ හෝ $x=2-\sqrt{3}$ 5

$\frac{\pi}{12} < \frac{\pi}{4}$

$\tan\frac{\pi}{12} < \tan\frac{\pi}{4}$

$\tan\frac{\pi}{12} < 1$

$\therefore \tan\frac{\pi}{12} = 2-\sqrt{3}$ 5

$\tan\frac{\pi}{12}\cdot\tan\frac{5\pi}{12} = 1$ (මෙම දෙකේ ගුණිතය) 5

$\boxed{45}$

(c)(i) $f(x) = 2\sqrt{3}\cos^2 x - \sin 2x$

$f(x) = 2\sqrt{3}\left(\frac{1+\cos 2x}{2}\right) - \sin 2x$ 5

$f(x) = \sqrt{3} + \sqrt{3}\cos 2x - \sin 2x$

$= \sqrt{3} + 2\left(\frac{\sqrt{3}}{2}\cos 2x - \frac{1}{2}\sin 2x\right)$ 10

$f(x) = \sqrt{3} + 2\left(\sin\frac{\pi}{3}\cos 2x - \cos\frac{\pi}{3}\sin 2x\right)$

$f(x) = \sqrt{3} + 2\sin\left(\frac{\pi}{3}-2x\right)$

$a=2,\ b=\sqrt{3},\ \alpha=\frac{\pi}{3}$ 5

(ii) $f(x) = \sqrt{3}+1$

$\sqrt{3} + 2\sin\left(\frac{\pi}{3}-2x\right) = \sqrt{3}+1$

$\sin\left(\frac{\pi}{3}-2x\right) = \frac{1}{2}$ 5

$\sin\left(\frac{\pi}{3}-2x\right) = \sin\frac{\pi}{6}$

$\frac{\pi}{3} - 2x = n\pi + (-1)^n\frac{\pi}{6}$ 5

$x = \frac{\pi}{6} - \frac{n\pi}{2} - (-1)^n\frac{\pi}{12}$; $n\in\mathbb{Z}^+$

(iii) $g(x) = f(x) - \sqrt{3} + 5$

$g(x) = \sqrt{3} + 2\sin\left(\frac{\pi}{3}-2x\right) - \sqrt{3} + 5$

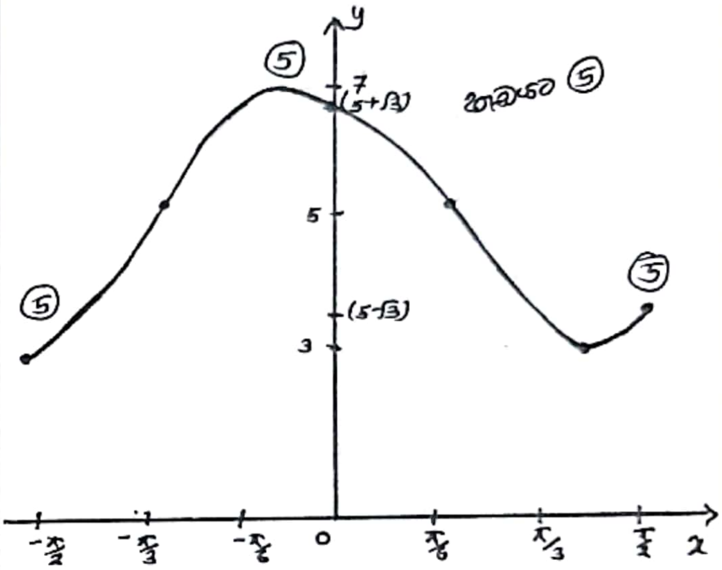
$g(x) = 2\sin\left(\frac{\pi}{3}-2x\right) + 5$

$\sin\left(\frac{\pi}{3}-2x\right) = \frac{g(x)-5}{2}$

$-1 \le \sin\left(\frac{\pi}{3}-2x\right) \le 1$

$-1 \le \frac{g(x)-5}{2} \le 1$

$3 \le g(x) \le 7$



$\boxed{50}$

(d) $3\sin A = 6\sin B = 2\sqrt{3}\sin C$

$\frac{\sin A}{2} = \frac{\sin B}{1} = \frac{\sin C}{\sqrt{3}} = k$

$\cos A = \frac{(\sqrt{3}k)^2+(k)^2-(2k)^2}{2\sqrt{3}k\cdot k}$ 10

$\cos A = 0$ 5

$A = \frac{\pi}{2}$ 5

$\boxed{20}$